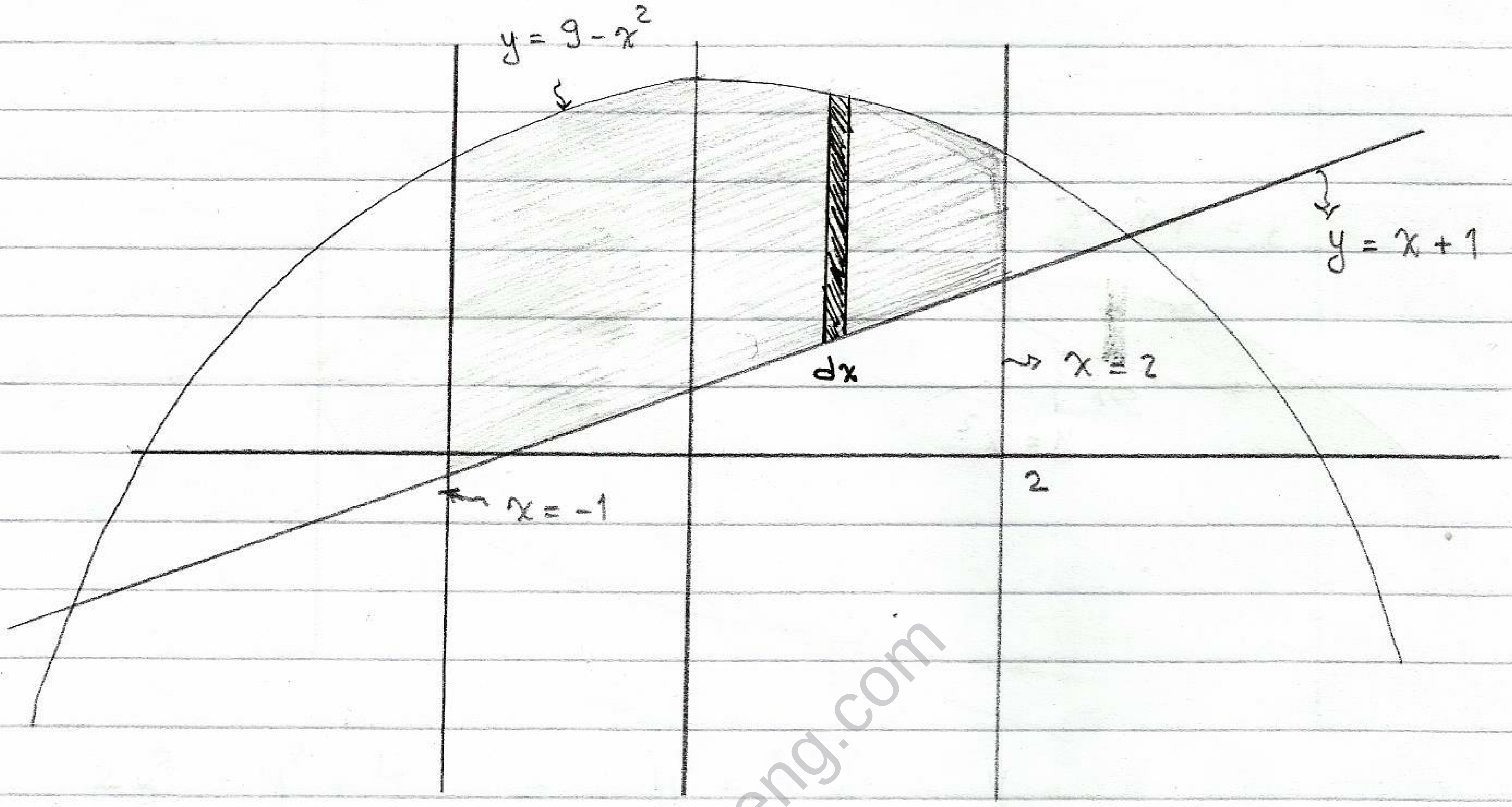
6.1 Assignment 1

Mostafa Ahmed Zein   group: 5   Section: 10

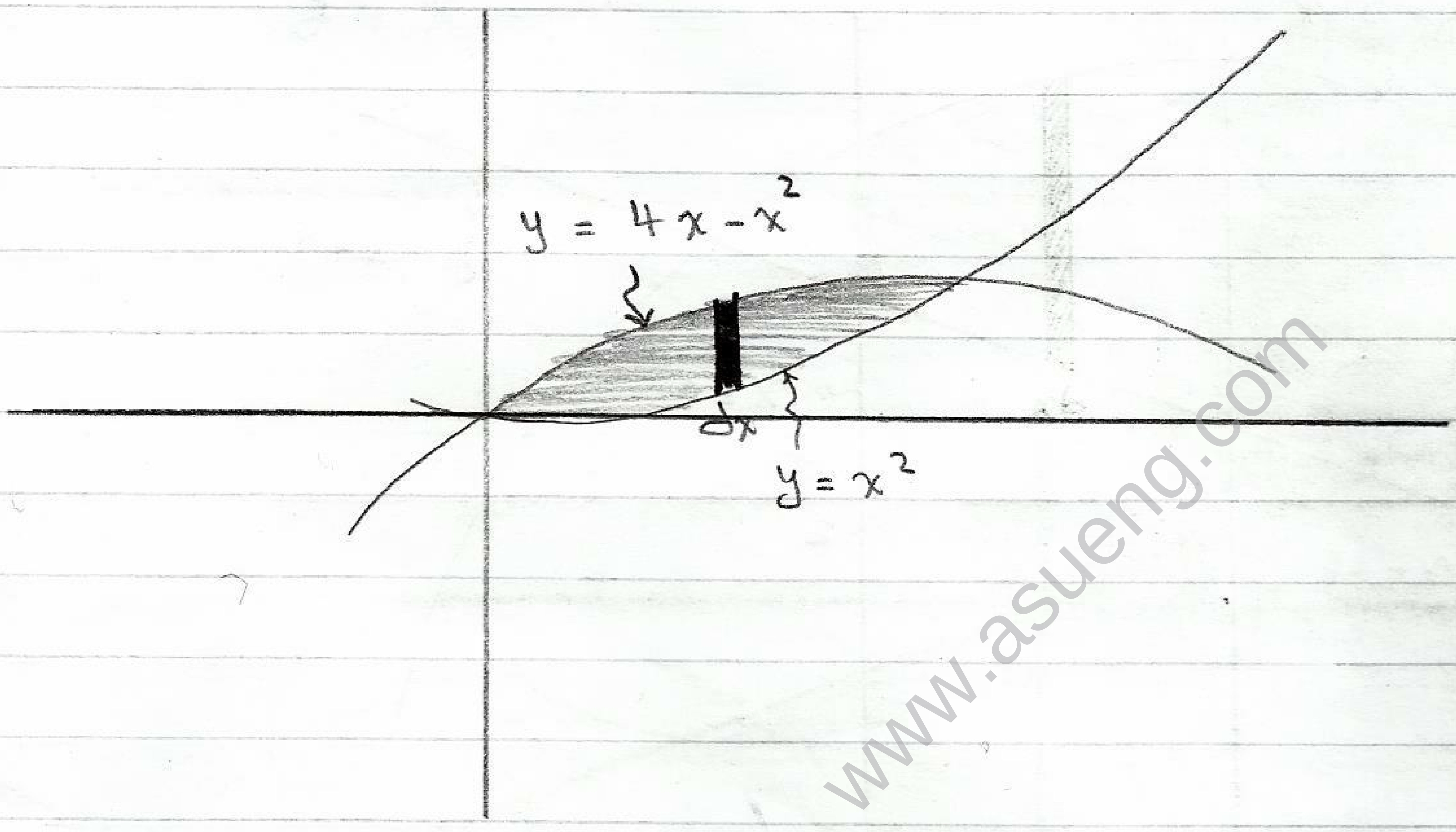
$\boxed{5}$ $y = x + 1$, $y = 9 - x^2$, $x = -1$, $x = 2$



$$\int_{-1}^{2} (9 - x^2) - (x + 1)\, dx = \int_{-1}^{2} -x^2 - x + 8 \;\; dx$$

$$= \left[\frac{-x^3}{3} - \frac{x^2}{2} + 8x \right]_{-1}^{2} = \frac{-8}{3} - 2 + 16 - \left(\frac{1}{3} - \frac{1}{2} - 8 \right) = 19.5$$

$\boxed{12}$ $y = x^2$, $y = 4x - x^2$

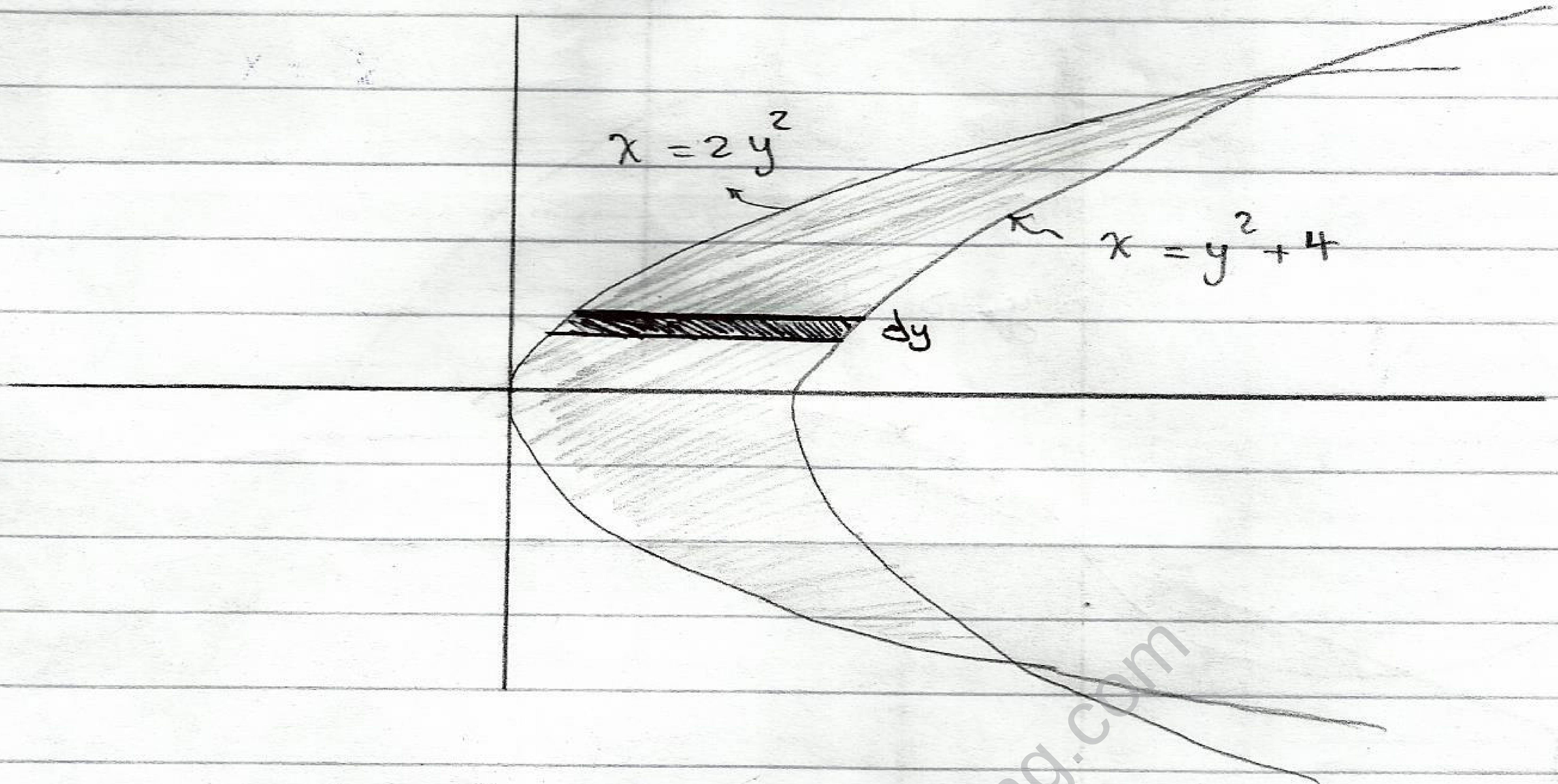


$x^2 = 4x - x^2$; $x = 0, 2$

$$\int_0^2 4x - x^2 - x^2 \, dx = \int_0^2 4x - 2x^2 \, dx$$

$$= 2x^2 - \frac{2}{3}x^3 \Big]_0^2 = 8 - \frac{16}{3} = \frac{8}{3}$$

19. $x = 2y^2$ $\qquad$ $x = y^2 + 4$



$2y^2 = y^2 + 4 \quad \therefore \quad y^2 = 4 \quad \therefore \quad y = \pm 2$

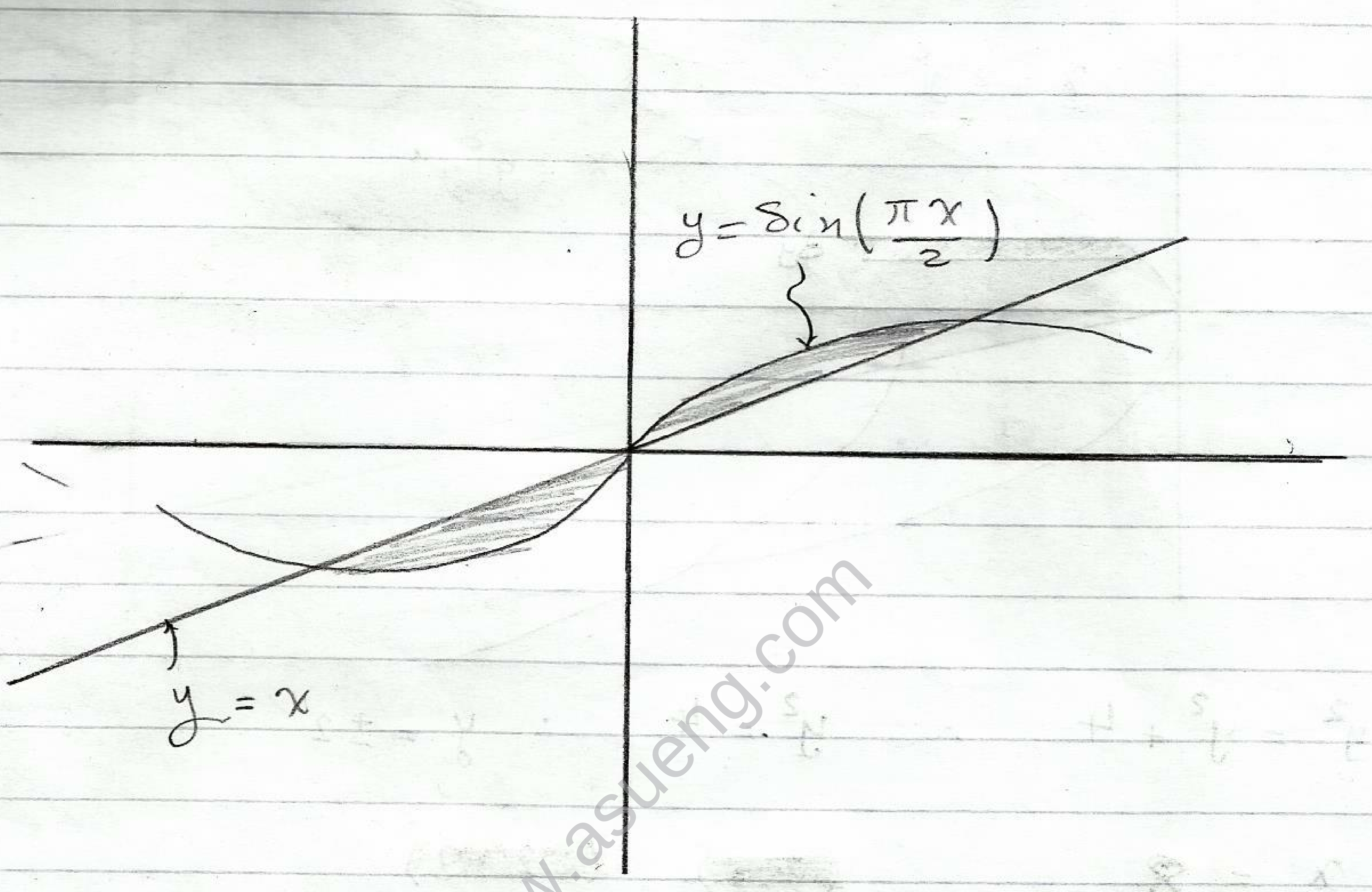
$\therefore x = 8$ [redacted]

$$\int_{-2}^{2} (y^2+4) - 2y^2 \, dy = \int_{-2}^{2} 4 - y^2 \, dy$$

$$= 2\int_{0}^{2} 4 - y^2 \, dy = 2\left(4y - y^3/3\right)\Big|_{0}^{2} = \frac{32}{3}$$

$\boxed{22}$ $y = \sin\left(\frac{\pi x}{2}\right)$

$y = x$



$$A = 2\int_0^1 \left[\sin\left(\frac{\pi x}{2} - x\right)\right] dx$$

$$= 2\left[\frac{-2}{\pi} \cos\left(\frac{\pi x}{2}\right) - \frac{x^2}{2} \right]_0^1$$

$$= 2\left[0 - \frac{1}{2}) - \left(\frac{-2}{\pi} - 0 \right) \right] = \frac{4}{\pi} - 1$$

6.2

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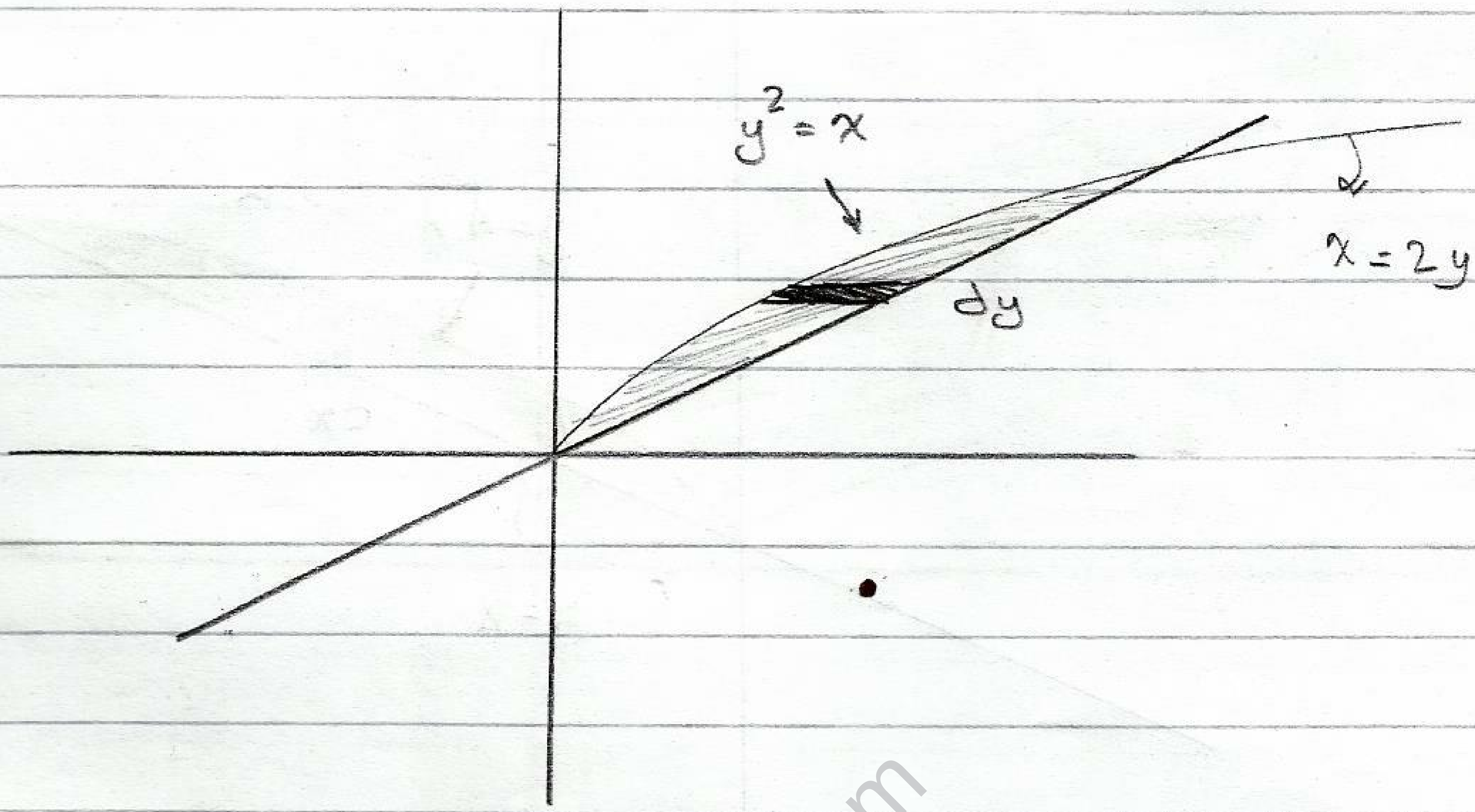
[9] $y^2 = x$, $x = 2y$ about the y-axis



$y^2 - 2y = 0$ $y(y-2) = 0$ $y = 0$ & 2

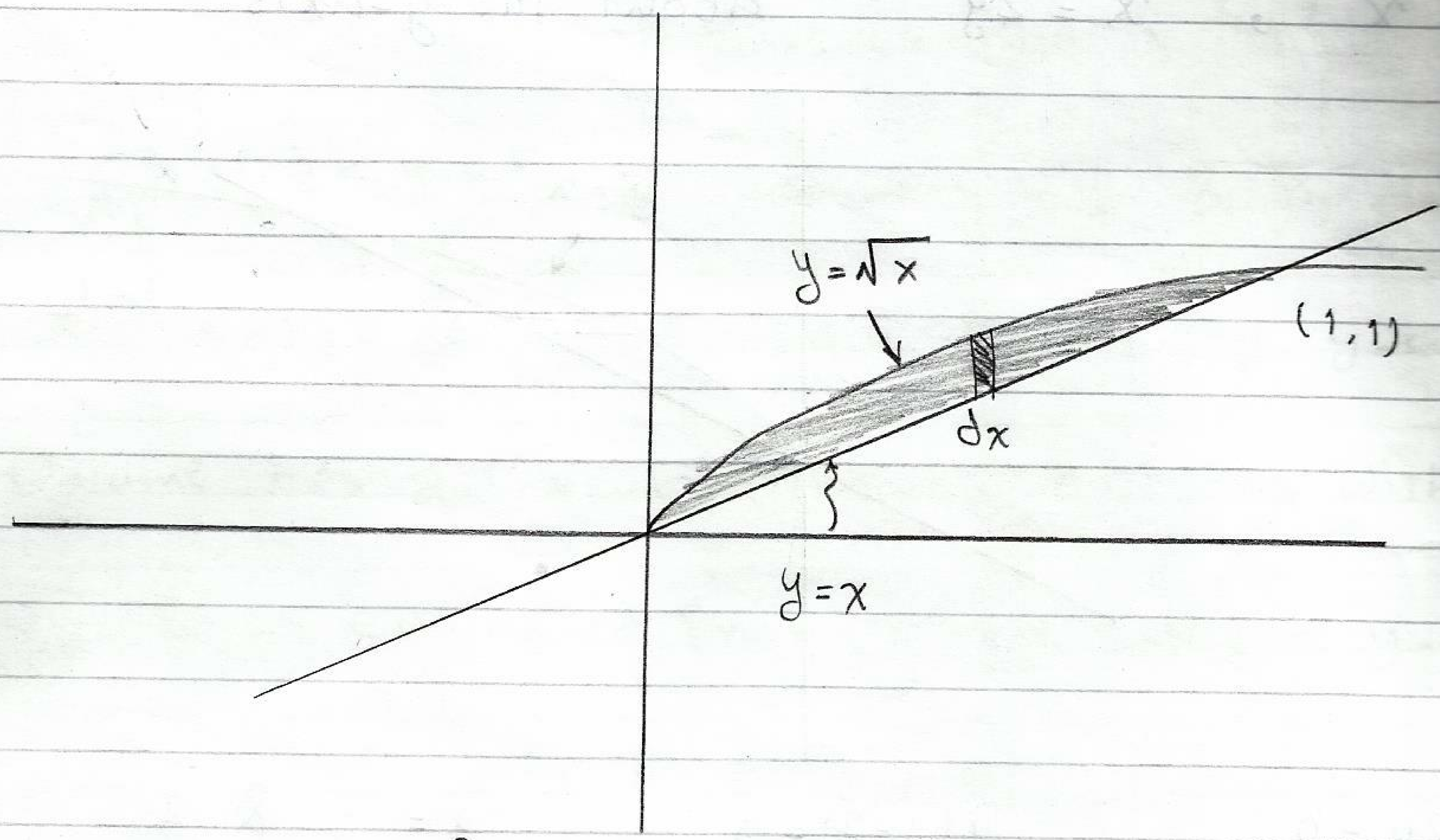
$$V = \pi \int_0^2 (2y - y^2)^2 \, dy$$

$$= \pi \int_0^2 4y^2 - y^4 \, dy$$

$$= \pi \left[\frac{4}{3}y^3 - \frac{y^5}{5} \right]_0^2$$

$$= \frac{64}{15}\pi$$

$\boxed{11}$ $y = x$, $y = \sqrt{x}$ about $y = 1$



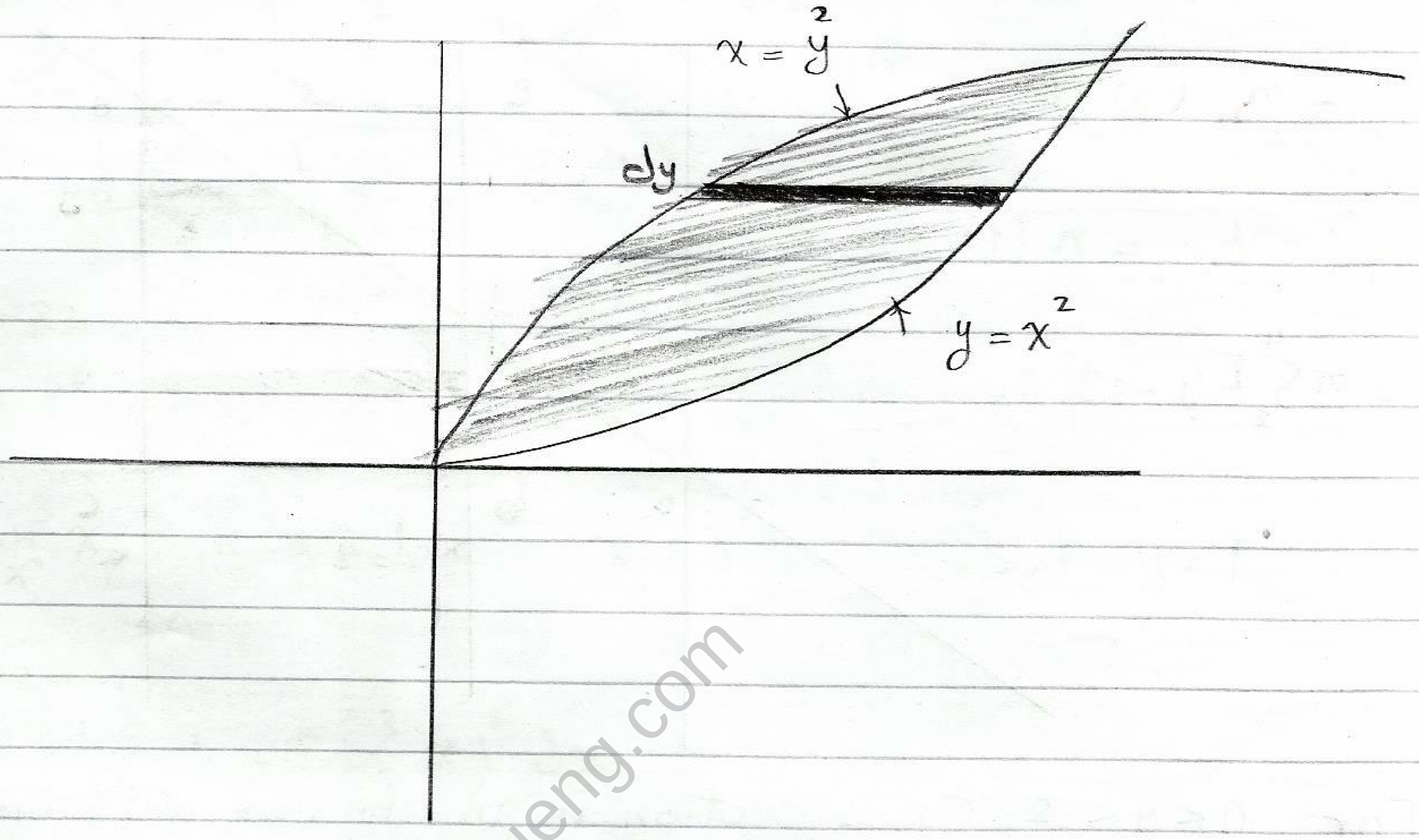
$$V = \pi \int_0^1 (1-x)^2 - (1-\sqrt{x})^2 \, dx$$

$$= \pi \int_0^1 x^2 - 2x + \cancel{1} - x + 2\sqrt{x} - \cancel{1} \, dx$$

$$= \left[\frac{x^3}{3} - x^2 - \frac{x^2}{2} + \frac{4}{3} y^{\frac{3}{2}} \right]_0^1 \pi$$

$$= \frac{\pi}{6}$$

$\boxed{17}$ $y = x^2$, $x = y^2$ about $x = -1$



$$V = \pi \int_0^1 (-1-\sqrt{y})^2 - (-1-y^2)^2 \, dy$$

$$= \pi \int_0^1 (-1-y^2)^2 - (-1-\sqrt{y})^2 \, dy$$

$$= \pi \int_0^1 y + 2\sqrt{y} \cancel{+1} - y^4 - 2y^2 \cancel{-1} \, dy$$

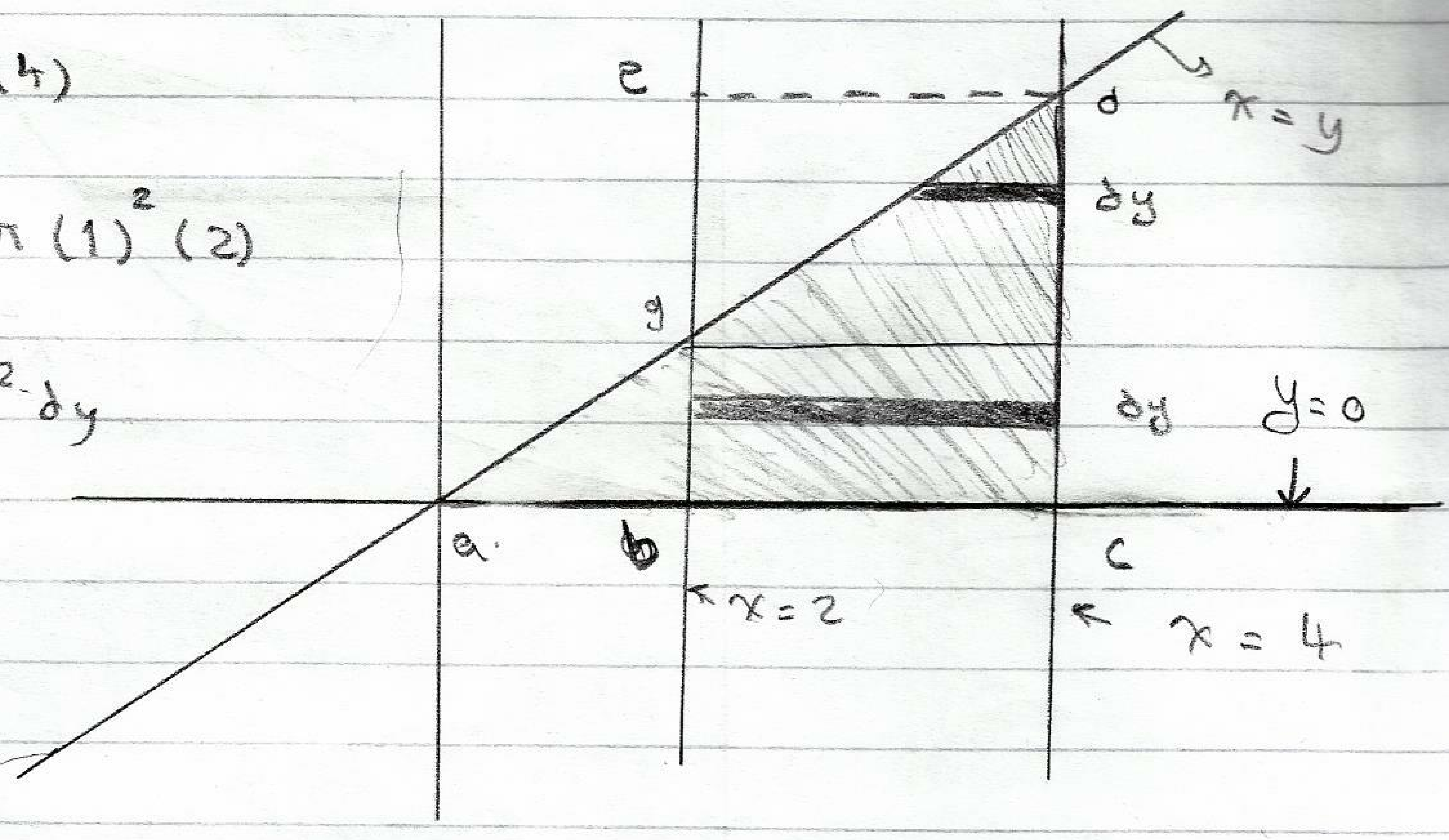
$$= \pi \left[\frac{y^2}{2} + \frac{4}{3} y^{\frac{3}{2}} - \frac{y^5}{5} - \frac{2}{3} y^3 \Big|_0^1 \right] = \frac{29}{30}\pi$$

(18) $y = x \qquad y = 0 \qquad x = 2 \qquad x = 4$ about $x = 1$

$V_{abcde} = \pi_4 (3)^2 (4)$

$V_{abgh} = V_{cyc} = \pi (1)^2 (2)$

$V = \pi \int_2^4 [y-1]^2 \, dy$



For $0 < y < 2$ Cross Section is an annulus with inner radius 2-1 & outter 4-1. The area is $A_1(y) = \pi(4-1)^2 - \pi(2-1)^2$. For $2 < y < 4$ The Cross Section Area will be the annulus radius equal to $y-1$ & outter radius $= 4-1$

The area is represented as $A_2(y) = \pi(4-1)^2 - \pi(y-1)^2$

$$V = \int_0^4 A(y) \cdot dy = \pi \int_0^2 [(4-1)^2 - (2-1)^2] \, dy + \pi \int_2^4 [(4-1)^2 - (y-1)^2] \, dy$$

$$= \pi [8y]_0^2 + \pi \int_2^4 (8 + 2y - y^2) \, dy$$

$$= 16\pi + \pi \left[8y + y^2 - \frac{1}{3}y^3\right]_2^4$$

$$= 16\pi + \pi \left[\left(32 + 16 - \frac{64}{3}\right) - \left(16 + 4 - \frac{8}{3}\right)\right] = \frac{76}{3}\pi$$

$$V = \pi \int_0^8 [(2 - \frac{1}{4}x)^2 - (2 - \sqrt[3]{x})^2]\, dx$$

$$= \pi \int_0^8 (-x + \frac{1}{16}x^2 + 4x^{\frac{1}{3}} - x^{2/3})\, dx$$

$$= \pi [-\frac{1}{2}x^2 + \frac{1}{48}x^3 + 3x^{4/3} - \frac{3}{5}x^{5/3}]_0^8$$

$$= \pi(-32 + \frac{32}{3} + 48 - \frac{96}{5}) = \frac{112}{15}\pi$$

31) $y = \tan^3 x \qquad y = 1 \qquad x = 0 \qquad$ about $y = 1$

$$V = \int_0^{\pi/4} (1 - \tan^3 x)^2\, dx$$

35) $$V = \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} ([3-(-2)]^2 - [\sqrt{y^2+1} - (-2)]^2\, dy$$

$$= \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} (5^2 - [\sqrt{y^2+1} - (-2)]^2)\, dy$$

41) $V = \pi \int_0^{\pi/2} \cos^2 x\, dx$ describes the Volume of the Solid obtained by rotating the region

$R = [(x, y) \| 0 \le x \le \frac{\pi}{2}, \quad 0 \le y \le \cos x]$

about the x-axis

(41) $\int_2^5 \pi \sqrt{y}^2 \, dy$ $\Rightarrow$ describes Volume based on a Circle's area $\int \pi r^2$

$r = \sqrt{y}$; $2 < y < 5$ $\Rightarrow$ radius is $\sqrt{y}$ & The Volume is evaluated between $2 < y < 5$ on thy y-axis

$\therefore$ The Volume has a Circular Cross-Sections with radius $\sqrt{y}$ & is evaluated from 2 to 5 on the y-axis

6.5

Mostafa Ahmed Zein Section: 10 Number: 21

$\boxed{5}$ $f(t) = t\sqrt{1+t^2}$ $[0,5]$

$$Ave = \frac{1}{5-0}\int_0^5 t\sqrt{1+t^2}\,dt \Rightarrow u = 1+t^2 \Rightarrow du = 2t\,dt$$

$$\therefore Ave = \frac{1}{5}\int_0^5 t\sqrt{u}\,\frac{du}{2t} \Rightarrow Ave = \frac{1}{10}\int_0^5 \sqrt{u}\,du \Rightarrow = \frac{1}{10}\left[\frac{2}{3}u^{\frac{3}{2}}\right]$$

$$\Rightarrow = \frac{1}{10}\left[\frac{2}{3}(1+t^2)^{\frac{3}{2}}\right]_0^5 = \frac{1}{30}\left[(1+t^2)^{\frac{3}{2}}\right]_0^5$$

$$= 8.804967157$$

$\boxed{7}$ $h(x) = \cos^4 x \sin x$ $[0,\pi]$ $\Rightarrow u = \cos x \Rightarrow du = -\sin x\,dx$

$$\therefore dx = \frac{-du}{\sin x} \Rightarrow Ave = \frac{1}{\pi}\int_0^\pi (\cos x)^4 \sin x\,dx$$

$$\Rightarrow \frac{1}{\pi}\int_0^\pi (u)^4 \cancel{\sin x}\,\frac{-du}{\cancel{\sin x}} \Rightarrow \frac{-1}{\pi}\int_0^\pi (u)^4\,du$$

$$\Rightarrow \frac{-1}{\pi}\left[\frac{u^5}{5}\right]_0^\pi \Rightarrow \frac{-1}{\pi}\left[(\cos x)^5\right]_0^\pi$$

$$\Rightarrow \frac{1}{5\pi} + \frac{1}{5\pi} = \frac{2}{5\pi}$$

$\boxed{8}$ $h(r) = \frac{3}{(1+r)^2}$ $[1,6]$

$$\frac{1}{6-1}\int_1^6 \frac{3}{(1+r)^2}\,dr \Rightarrow 1+r = u \Rightarrow du = dr$$

$$\xRightarrow{Ave} \frac{1}{5}\int_1^6 \frac{3}{u^2}\,du \xRightarrow{Ave} \frac{1}{5}\int_1^6 3u^{-2}\,du \Rightarrow \frac{1}{5}\left[-3u^{-1}\right]_1^6$$

$$= \frac{1}{5}\left[-3(1+r)^{-1}\right]_1^6 = \frac{3}{14} = 0,21428$$

$\boxed{10}$ (a) $f(x) = \sqrt{x}$ $\quad [0, 4]$

$$\frac{1}{4}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{4}{3}$$

(b) $\frac{4}{3} = \sqrt{c}$ $\quad \therefore c = \frac{16}{9} = 1,78$

(c) $\int_0^4 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{16}{3}$

rectangle dimensions is $\frac{4}{3}$ & 4

$\boxed{11}$ $f(x) = 2\sin x - \sin 2x$ $\quad [0, \pi]$

$$Ave = \frac{1}{\pi}\int_0^\pi 2\sin x - \sin 2x\,dx = \frac{1}{\pi}\left[-2\cos x + \frac{1}{2}\cos 2x\right]_0^\pi$$

$$= \frac{5}{2\pi} + \frac{3}{2\pi} = \frac{4}{\pi}$$

(b) $2\sin c - \sin 2c = \frac{4}{\pi}$

$\therefore 2\sin c - 2\sin c\cos c = \frac{4}{\pi}$

$\therefore 2\sin c\,(1 - \cos c) = \frac{4}{\pi}$

$\frac{3}{5}$

13) f is Continuous on $[1,3]$, by Mean Value theorem

$\therefore 8 = 2f(c) \quad \therefore f(c) = 4$

14) Ave $= \frac{1}{b} \int_0^b 2 + 6x - 3x^2 \, dx$

$Ave = \frac{1}{b}\left[2x + 3x^2 - x^3\right]_0^b = \frac{1}{b}\left[2b + 3b^2 - b^3\right]$

$\therefore 2 + 3b - b^2 = 3 \quad \therefore -b^2 + 3b - 1 = 0$

$\therefore b^2 - 3b + 1 = 0 \quad \therefore b = 2.618$ or $b = 0.381966$

Review Chapter 6

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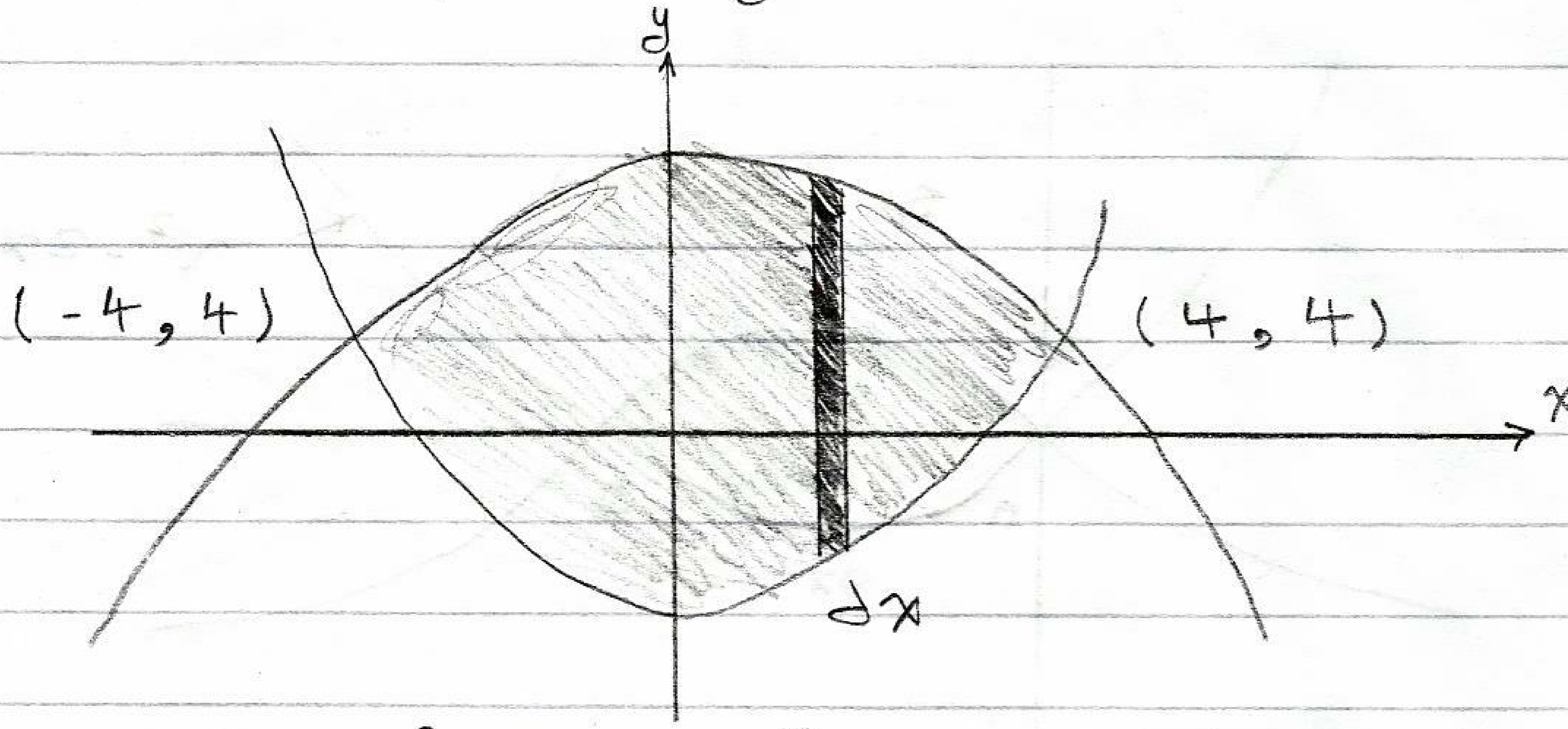
[2] $y = 20 - x^2$, $y = x^2 - 12$

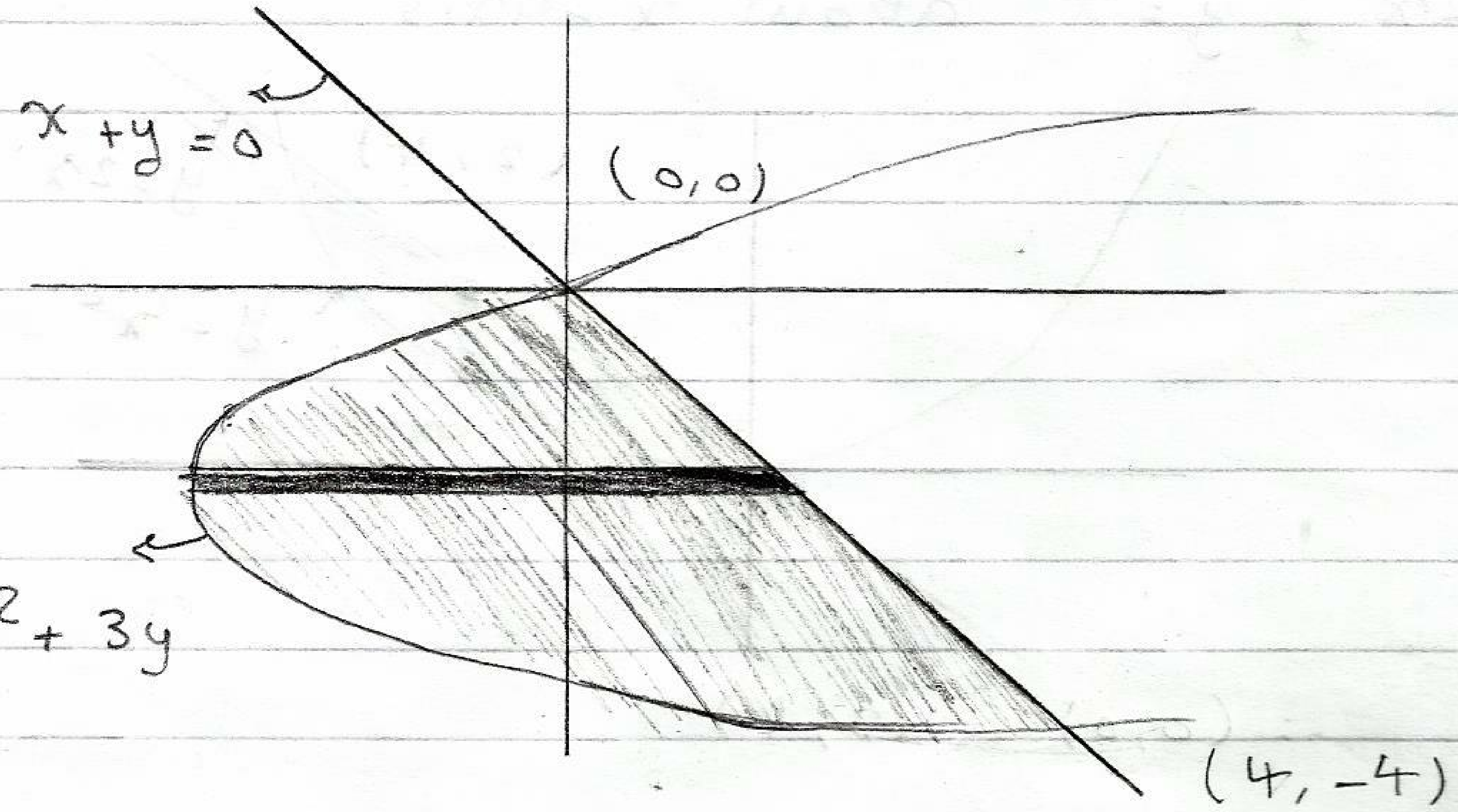


$x^2 - 12 = 20 - x^2$, $2x^2 = 32$, $16 = x^2$ $\therefore x = \pm 4$

$A = \int_{-4}^{4} (20 - x^2 - x^2 + 12)\,dx = \int_{-4}^{4} (-2x^2 + 32)\,dx$

$= \left[\frac{-2}{3} x^3 + 32x \right]_{-4}^{4} = \frac{512}{3}$

[4] $x + y = 0$ $x = y^2 + 3y$

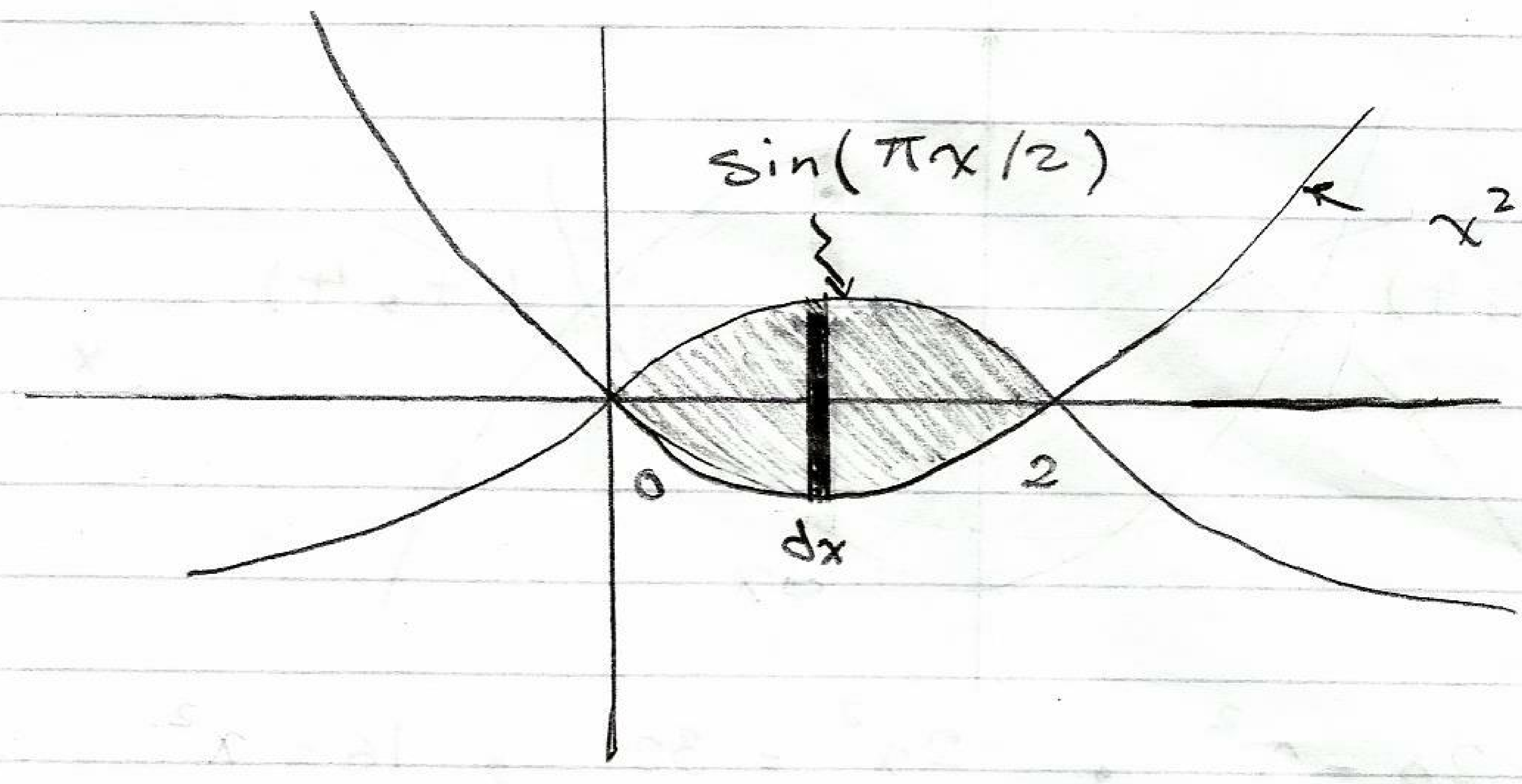


$-y = y^2 + 3y \Rightarrow y^2 + 4y = 0 \Rightarrow y(y+4) = 0 \Rightarrow \therefore y = 0 \ \& \ -4$

$\therefore (0,0) \ \& \ (4,4) \Rightarrow A = \int_{-4}^{0} y^2 + 3y + y \ dy \Rightarrow = \int_{-4}^{0} y^2 + 4y \ dy$

$= \left[\frac{y^3}{3} + 2y^2 \right]_{-4}^{0} = \frac{32}{3}$

5 $y = \sin(\pi x/2)$ $\quad\quad y = x^2 - 2x$

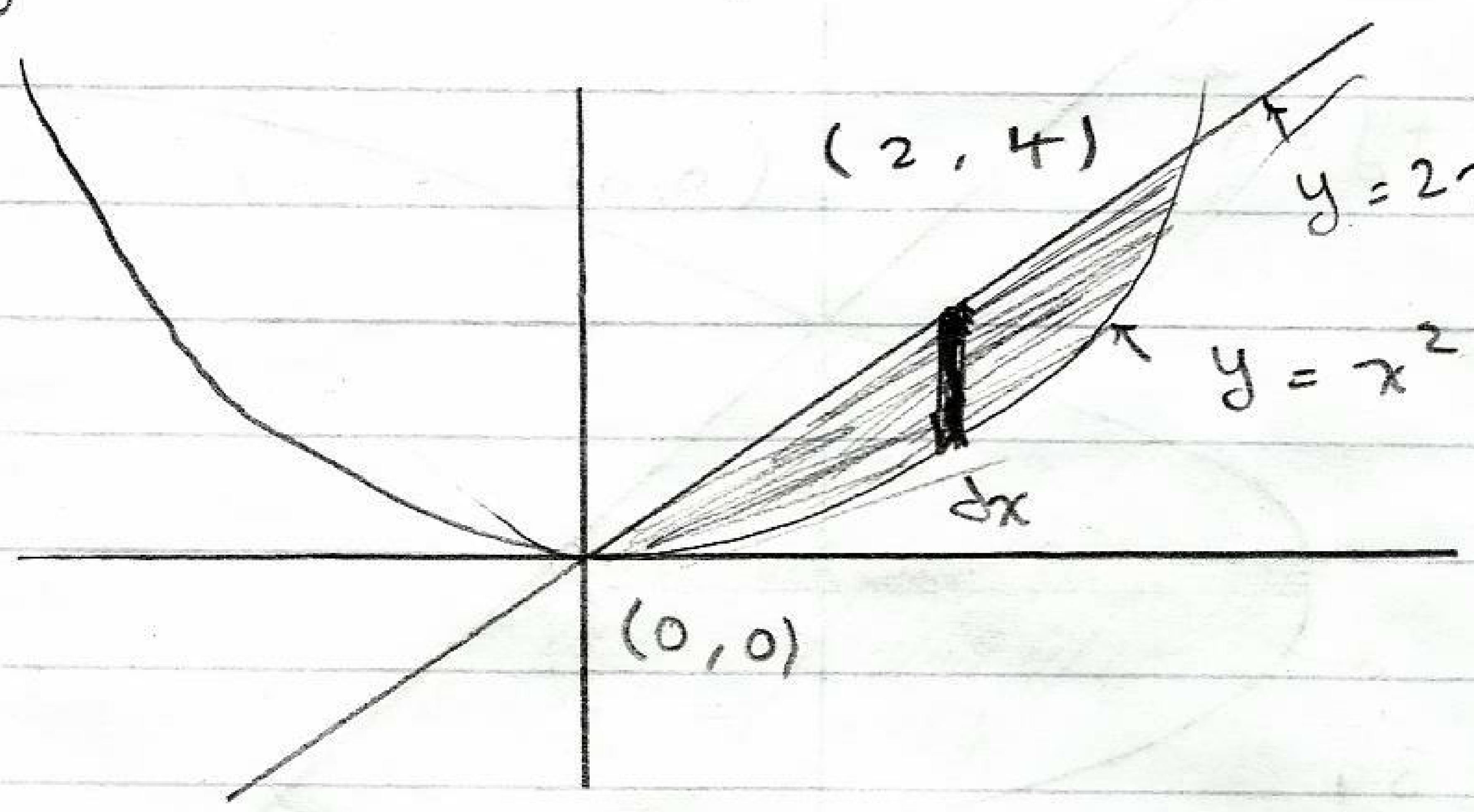


$x^2 - 2x = \sin(\pi x/2) \quad \therefore (0,0) \;\&\; (2,0)$

$A = \int_0^2 \sin\left(\frac{\pi x}{2}\right) - x^2 + 2x \, dx \Longrightarrow = \left[\frac{-2}{\pi} \cos\left(\frac{\pi x}{2}\right) - \frac{x^3}{3} + x^2 \right.$

$= \frac{4}{3} + \frac{4}{\pi} \simeq 2,6065728$

7 $y = 2x$, $y = x^2$ about x-axis

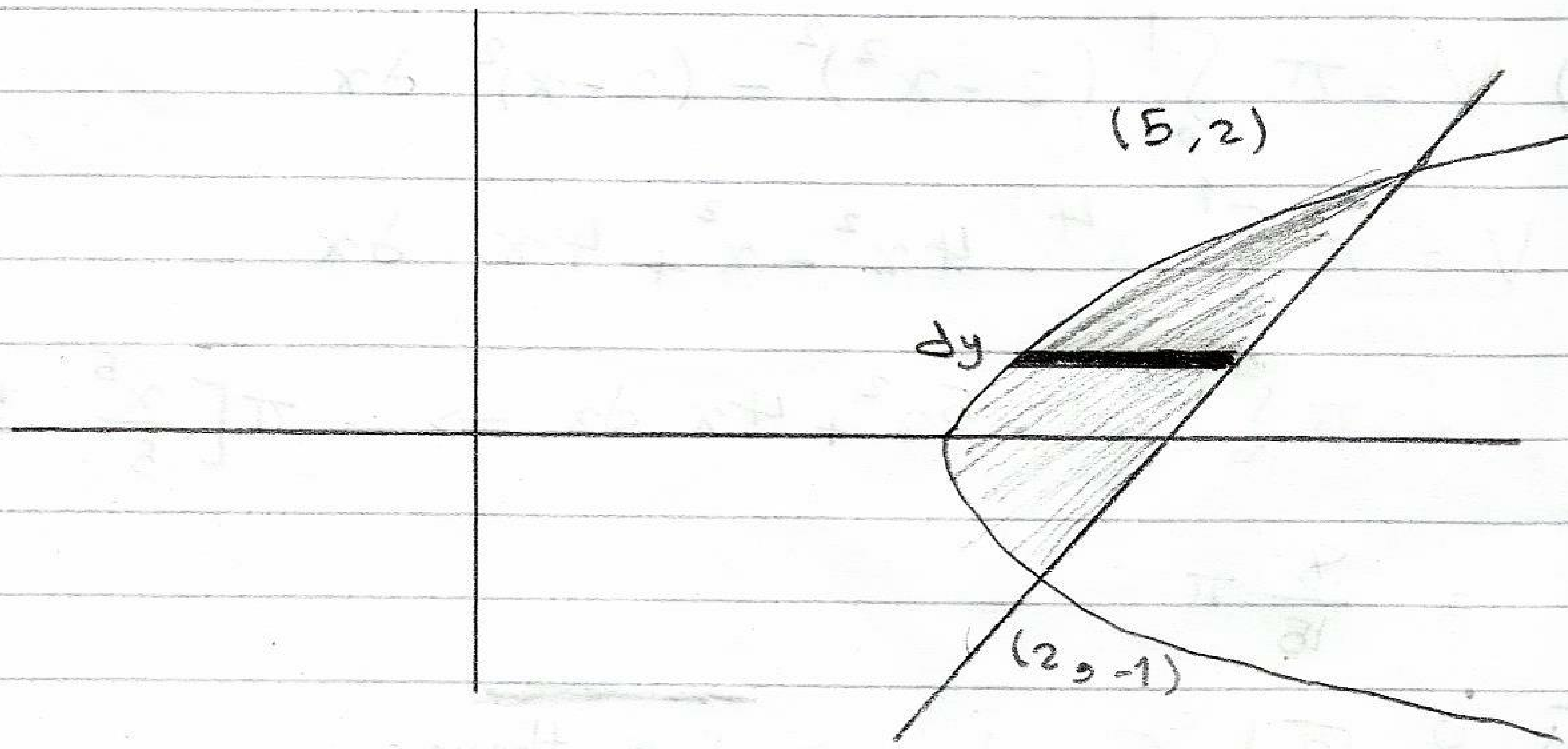


$x^2 = 2x \quad \therefore (0,0) \;\&\; (2,4)$

$\therefore V = \pi \int_0^2 (2x)^2 - (x^2)^2 \, dx \Longrightarrow = \pi \int_0^2 4x^2 - x^4 \, dx$

$= \pi \left[\frac{4}{3} x^3 - \frac{x^5}{5} \right]_0^2 = \frac{64\pi}{15}$

$\boxed{8}$ $x = 1 + y^2$, $y = x - 3$ about the y-axis



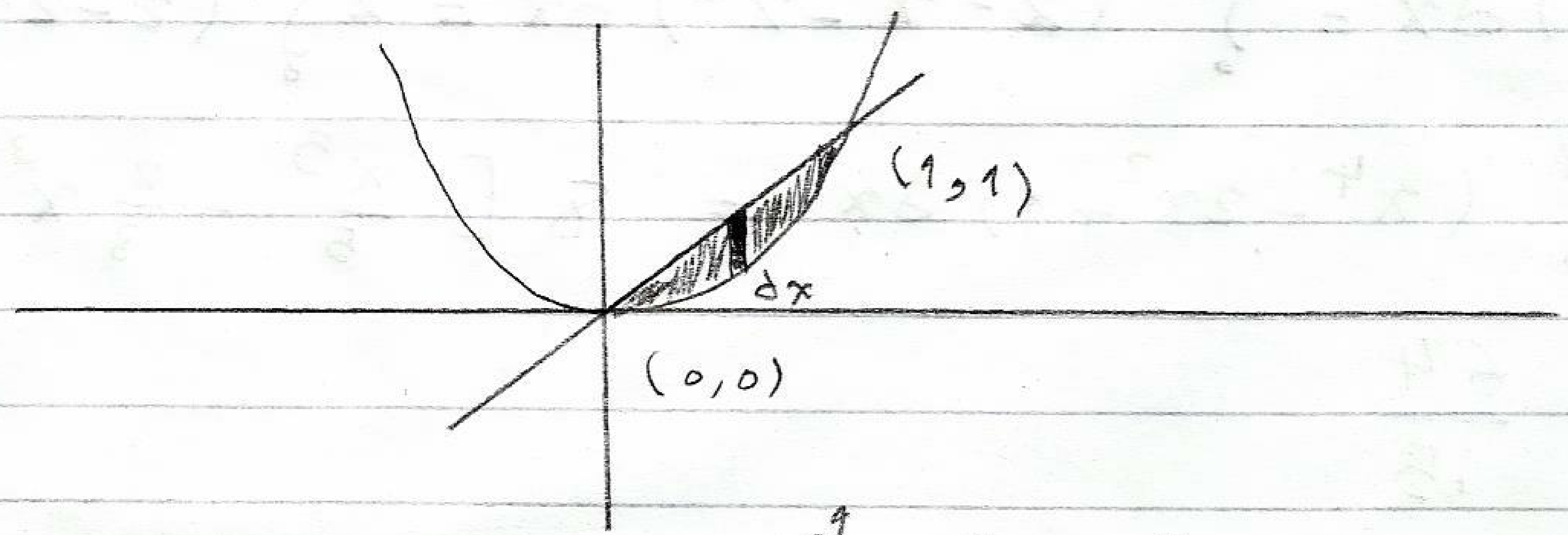
$y + 3 = 1 + y^2 \therefore y^2 - y - 2 = 0 \quad \therefore y = 2 \ \& -1 \quad \therefore (5,2) \ \& \ (2,-1)$

$$V = \pi \int_{-1}^{2} (y+3)^2 - (1+y^2)^2 \, dy$$

$$= \pi \int_{-1}^{2} y^2 + 6y + 9 - y^4 - 2y^2 - 1 = \pi \int_{-1}^{2} -y^4 - y^2 + 6y + 8 \, dy$$

$$= \pi \left[\frac{-y^5}{5} - \frac{y^3}{3} + 3y^2 + 8y \right]_{-1}^{2} = \pi \left[\frac{284}{15} + \frac{67}{15} \right] = \frac{117}{5}$$

$\boxed{15}$ $y = x$, $y = x^2$



(a) The x-axis $\Rightarrow V = \pi \int_0^1 x^2 - x^4 \, dx$

$$\Rightarrow \; = \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1 = \frac{2}{15}\pi$$

(b) The y-axis $\Rightarrow V = \pi \int^1 (\sqrt{y})^2 - y^2 \, dy$

$$V = \pi \int_0^1 (y - y^2)\, dy \Rightarrow = \pi \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = \frac{\pi}{6}$$

(c) $V = \pi \int_0^1 (2-x^2)^2 - (2-x)^2 \, dx$

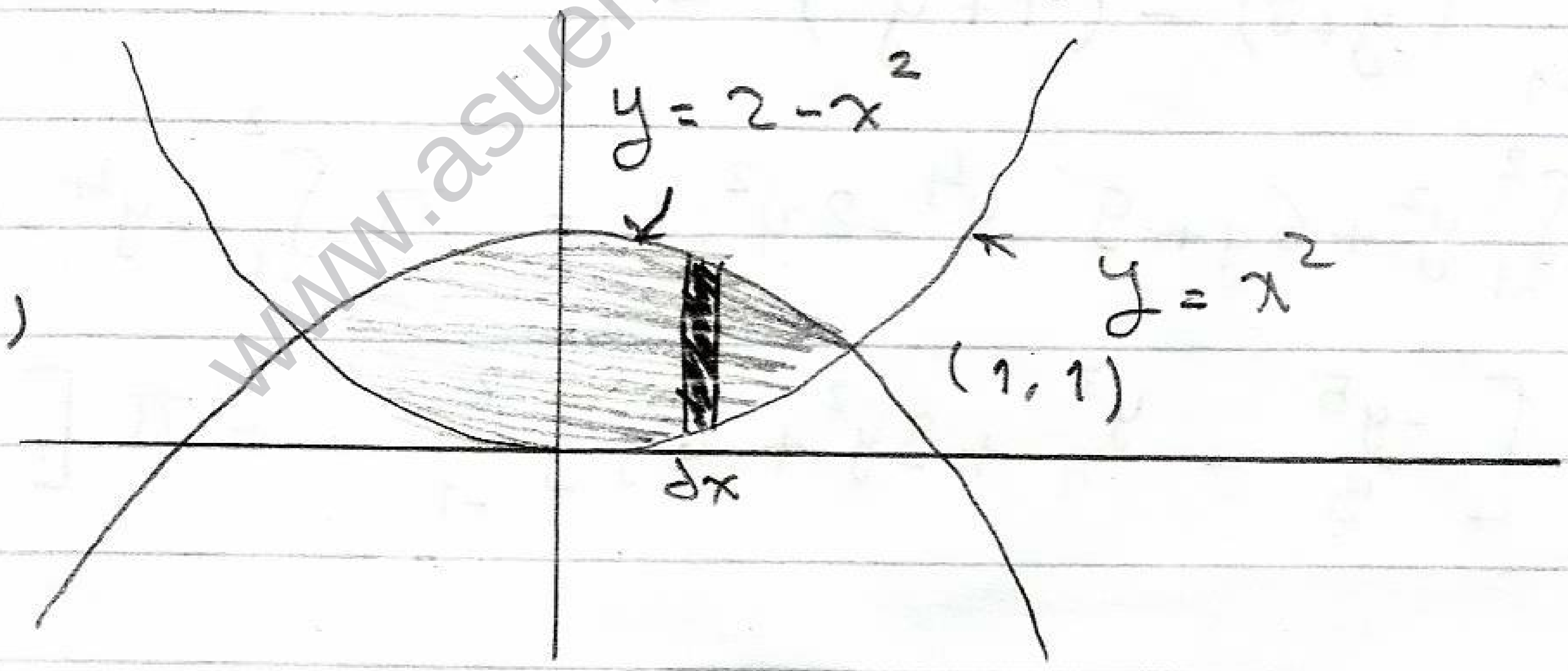
$$V = \pi \int_0^1 x^4 - 4x^2 - x^2 + 4x \, dx$$

$$= \pi \int_0^1 x^4 - 5x^2 + 4x \, dx \Rightarrow = \pi \left[\frac{x^5}{5} - \frac{5}{3}x^3 + 2x^2 \right]_0^1$$

$$= \frac{8}{15}\pi$$

19 & 20 Couldn't Solve them

24 $y = x^2$, $y = 2 - x^2$



$$V = A(x)\, dx = \int_0^1 (2 - x^2 - x^2)\, dx = 2 \int_0^1 (2 - 2x^2)^2 \, dx$$

$$= 8 \int_0^1 (x^4 - 2x^2 + 1)\, dx = 8 \left[\frac{x^5}{5} - \frac{2}{3}x^3 + x \right]_0^1$$

$$= \frac{64}{15}$$

30 $f(t) = t \sin(t^2)$ $[0, 10]$ $u = t^2$, $du = 2t\, dt$

$$\therefore AV = \frac{1}{10} \int_0^{10} t \sin(t^2)\, dt = \frac{1}{20} \int_0^{10} \cancel{t} \sin(u) \frac{du}{2\cancel{t}}$$